Online parameter estimation of adaptive cruise control models with delays and lags

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***Abstract*— In this work we consider online parameter estimation of *adaptive cruise control* (ACC) equipped vehicles which may contain time delays (e.g., sensor delays) and lags (e.g., actuator lag). We extend a *recursive least squares* (RLS) method and apply it to calibrate models with various magnitudes of delay and lag, and study the performance of the method when measurement noise is present. We show that RLS can be applied to exactly recover model parameters given noise-free measurement data. In addition, the RLS estimator performs well under typical sensor noises associated with onboard sensors such as GPS and radars. The method is tested on data from a 2019 ACC-equipped vehicle, and the results show that the overall quality of fit via RLS is comparable to a commonly used batch optimization method (4% and 0.8% mean absolute error on the space gap and the velocity profiles, respectively). RLS is shown to run two orders of magnitude faster than the batch optimization method.**

I. Introduction

As one of the most important driver-assistance features, *adaptive cruise control* (ACC) has gained significant research attention over the past several decades [1]–[4]. Originally developed to study human driving behaviors [5], car-following models are now applied to ACC systems as measurement data from on-board sensors become available (e.g., GPS, radar, lidar etc.).

This work considers three commonly used models that describe an ACC dynamical system. The simplest of these is the *constant-time headway relative velocity* (CTH-RV) model considered in [4], [6]–[8]. The CTH-RV model is an *ordinary differential equation* (ODE) that models the second-order dynamics (acceleration) of the vehicle:

$$\dot{v}(t) = f(s(t), v(t), \Delta v(t)), \tag{1}$$

where f is a linear function with arguments $s(t)$ the space gap between the ACC vehicle and the vehicle in front, $v(t)$ the velocity of the ACC vehicle, and $\Delta v(t)$ the relative velocity between the two vehicles, all being time-varying quantities. Several studies [1], [2], [9], [10] point out additional practical considerations for modeling ACC systems, including the presence of systematic delays caused by sampling of on-board sensors and the actuator lags, for example. As a result, the second-order linear model described by the CTH-RV can be extended to include actuator lag via third order dynamics [1], [2], [9], as well as sensor delay via a delay differential equation [4].

It is important to consider the possible time-delays and lags when calibrating ACC models from noisy measurement data, because they have a potentially negative impact on smoothing traffic waves [9], [10]. Correctly identifying the delays can consequently help to develop anticipatory strategies to compensate the effects on global traffic [10].

Efforts for finding model parameters, given measurement data (often called an *inverse problem* [11]), generally are done using *batch optimization* techniques. For example, Kesting and Treiber [12], Punzo et al. [13] and Gunter et al. [4], [7] formulate the problem as a search for parameters that minimize error between simulated trajectories and the measured ones. The task of finding the parameters of dynamical models can exhibit many challenges, mostly due to ill-conditioning and non-convexity [14], [15]. The parameter error convergence is greatly dependent upon the frequency information contained in the input signal [16], and the solution to a global minimum sometimes is not guaranteed. Despite these challenges, only a few studies have considered the *practical parameter identifiability* [17], [18], which determines whether the model parameters can be uniquely recovered given input and output measurements of a car-following system. For example, Monteil et al. [19] and Punzo et al. [20] conduct a sensitivity-based identifiability analysis before estimating the parameters associated with a car-following model given the captured driving behaviors. Our previous work [21] demonstrates that at certain regimes (equilibrium driving, for instance), the model parameters are not uniquely identifiable. Another challenge for the inverse problem is that sensor measurements are sometimes noisy. It is also important to correctly characterize measurement noise in the estimation routine [22].

This work considers the practical challenges of parameter estimation of an ACC driving system. We extend the *recursive least squares* (RLS) applied in our previous work [21] to the CTH-RV with delays and lags. We investigate the accuracy of model calibration under a range of driving regimes and demonstrate the effect of sensor noise, time delays and lags on calibrating an ACC system. As a practical illustration, we also implement this method on the ACC radar data collected in [21].

The remainder of the work proceeds as follows. Section II

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briefly introduces three commonly used ACC models and presents the recursive least squares formulation for calibrating delay-free CTH-RV. In Section III, we extend the RLS estimator to account for parameter estimation of the actuator-lag and the sensor-delay models. In Section IV, we investigate model recovery using RLS with respect to the captured driving behaviors and measurement noise levels. In Section V, we present the results of model parameter estimation on the CAN bus data of a 2019 ACC equipped vehicle collected in [21]. The results show that RLS and its extension to models with lags and delays is able to accurately recover the parameters of all the ACC models considered in this work.

II. Preliminaries

A. Model definition

In this work we focus on the *constant-time headway relative velocity* model introduced in previous works [4], [7]:

$$\begin{aligned}\dot{s}(t) &= \Delta v(t)\\ \dot{v}(t) &= f(s(t), v(t), \Delta v(t))\\ &= k_1(s(t) - \tau_h v(t)) + k_2(\Delta v(t)).\end{aligned} \tag{2}$$

Here the space gap $s(t)$ is the distance between the ACC vehicle and the vehicle in front, and the dynamic is defined as the velocity difference between the two vehicles $\Delta v(t)$; the acceleration of the ACC vehicle $\dot{v}(t)$ is a linear function with respect to $s(t) - \tau_h v(t)$, the difference between $s(t)$ and the desired space gap $\tau_h v(t)$, parameterized by the desired constant-time headway that the ACC tries to maintain, τ_h, and $\Delta v(t)$. k_1 and k_2 are parameters representing the gains on both terms.

Model (2) can be extended to the third order (acceleration) dynamic to account for the actuator lag [1], [2]:

$$\begin{aligned}\dot{s}(t) &= \Delta v(t)\\ \dot{v}(t) &= a(t)\\ \dot{a}(t) &= \tfrac{1}{\tau_a}(f(s(t), v(t), \Delta v(t)) - a(t))\end{aligned} \tag{3}$$

where τ_a refers to the lag on the actuator.

The introduction of delay term can also be considered to account for the systematic delay in the sensors [23], [24]. Therefore, the explicit delay τ is added to modify the model (2) into:

$$\begin{aligned}\dot{s}(t) &= \Delta v(t)\\ \dot{v}(t) &= f(s(t-\tau), v(t-\tau), \Delta v(t-\tau))\end{aligned} \tag{4}$$

Note that (4) is a second-order *delay differential equation* (DDE) as opposed to a third-order ODE considering an actuator lag in (3). Therefore, the two distinct dynamical models result in different calibration routines, which we will address in detail in Section III.

B. Recursive least squares

In our previous work [21], we formulated a recursive least squares problem to identify the parameters of model (2) in an online setting. The constant-parameter estimation problem can be written as solving γ in

$$Y = X\gamma, \tag{5}$$

where $X \in \mathbb{R}^{m\times n}$ and $Y \in \mathbb{R}^m$ are data points organized such that the linear relationship between X and Y can be discovered. In a dynamical system such as an ODE, X and Y are usually tall matrices with time-series data, i.e., $m > n$. This results in an over-determined system, and the existence of a solution to γ is not guaranteed. In this case, we seek an approximate solution such that the mean squared error $\|Y - X\gamma\|_2^2$ is minimized. The general least squares approximation solution can be written as:

$$\hat{\gamma} = (X^T X)^{-1} X^T Y. \tag{6}$$

A recursive least squares update of parameter $\hat{\gamma}_k$ in (6) is implemented such that:

$$\hat{\boldsymbol{\gamma}}_k = \hat{\boldsymbol{\gamma}}_{k-1} + R_k^{-1} X_k (Y_k - X_k \hat{\gamma}_{k-1})^{-1}, \tag{7}$$

where $R_k = \sum_{i=1}^{k} X_i X_i^T$ is the cumulative outer product of X_k. Inversion of R_k at every timestep can be cumbersome to compute. For implementation, it is recommended to use the matrix inversion Lemma [17].

Next in Section III, we extend the RLS considered in [21] to recover the actuator lag model in the form of (3) and the sensor-delay model such as (4). A complete description and analysis is provided in [21].

III. RLS application on models with delays

Considering the parameters in models (3) and (4) are time-invariant, and the left-hand-side of the each of the dynamical models can be expressed as a linear combination of the parameter set if the time-series data of v, s, Δv and a are available. We take the advantage of this fact to formulate the model parameters estimation problem as a least squares regression problem.

A. Actuator lag CTH-RV model calibration

First we rewrite the continuous time ODE (3) in discrete-time using a forward Euler step scheme:

$$a_{k+1} = a_k + \frac{1}{\tau_a}\left(k_1(s_k - \tau_h v_k) + k_2(\Delta v_k) - a_k\right)\Delta T, \tag{8}$$

where a_k, s_k, v_k and Δv_k are the acceleration of the following vehicle, the space gap between the follower and the leader, the velocity of the following vehicle, and the velocity difference between the two cars, respectively. ΔT is the timestep size that is used to discretize the ODE (3) and is typically on the order of 1/10 sec in accordance with the sampling rate of some common on-board sensor platforms including the experiments presented later in this work.

The discrete-time model (8) can be rewritten as:

$$a_{k+1} = \gamma_1 v_k + \gamma_2 \Delta v_k + \gamma_3 s_k + \gamma_4 a_k, \quad (9)$$

where $\gamma_1 := -\frac{k_1 \tau_h}{\tau_a}\Delta T$, $\gamma_2 := \frac{k_2}{\tau_a}\Delta T$, $\gamma_3 := \frac{k_1}{\tau_a}\Delta T$ and $\gamma_4 := 1 - \frac{1}{\tau_a}\Delta T$. In order to estimate all four parameters, the acceleration data is required. We now demonstrate that one can recover $\boldsymbol{\gamma} := [\gamma_1, \gamma_2, \gamma_3, \gamma_4]^T$ from an experimental dataset containing $(v_k, \Delta v_k, s_k, a_k)$ for all $k \in \{1, ..., K\}$, via least-squares. We expand (9) in time by stacking the uniformly sampled measurements to obtain:

$$\begin{bmatrix} a_2 \\ a_3 \\ \vdots \\ a_K \end{bmatrix} = \begin{bmatrix} v_1 & \Delta v_1 & s_1 & a_1 \\ v_2 & \Delta v_2 & s_2 & a_2 \\ \vdots & \vdots & \vdots & \vdots \\ v_{K-1} & \Delta v_{K-1} & s_{K-1} & a_{K-1} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \\ \gamma_4 \end{bmatrix}, \quad (10)$$

which is the same form as (5). The vector Y contains the values of a_k from timestep 2 to the length of the data K. The data matrix X contains measurements of v_k, Δv_k, s_k and a_k from timestep 1 to $K-1$ in a column-wise order. The recursive implementation follows the same procedure outlined in (7).

B. Actuator lag CTH-RV model calibration

In the case of sensor delay, fitting a RLS estimator can be slightly less straightforward. We first rewrite the continuous time DDE (4) in discrete-time using a forward Euler step scheme:

$$v_{k+1} = v_k + k_1(s_{k-l} - \tau_h v_{k-l})\Delta T + k_2(\Delta v_{k-l})\Delta T, \quad (11)$$

with l being the number of timesteps of delay ($l\Delta T = \tau$). The assumption that the sensor delay τ is a multiple of the sampling size ΔT greatly simplifies the search space, which allows the RLS to enumerate on a finite set of possible delays instead of searching for the entire parameter space where the delay term might cause non-convexity. For the DDE model (4), the delay term τ cannot be found directly along with other parameters via a single run of RLS on (11). Instead, we decompose the problem into multiple variants of the least squares problem, each with a fixed l time-steps of delay. I.e., we solve γ for each possible l within a range (typically between 0-0.8 sec), and find the l that minimizes the error on the space gap. The corresponding sensor delay will be $\tau = l\Delta T$.

Again, we write the corresponding linear expression of (11) as:

$$v_{k+1} = v_k + \gamma_1 v_{k-l} + \gamma_2 \Delta v_{k-l} + \gamma_3 s_{k-l}, \quad (12)$$

with the γ's redefined as $\gamma_1 := -k_1 \tau_h \Delta T$, $\gamma_2 := k_2 \Delta T$ and $\gamma_3 := k_1 \Delta T$. The data is organized in correspondence of $Y = X\boldsymbol{\gamma}$ as:

$$\begin{bmatrix} v_{l+2} - v_{l+1} \\ v_{l+3} - v_{l+2} \\ \vdots \\ v_{k+1} - v_k \\ \vdots \\ v_K - v_{K-1} \end{bmatrix} = \begin{bmatrix} v_1 & \Delta v_1 & s_1 \\ v_2 & \Delta v_2 & s_2 \\ \vdots & \vdots & \vdots \\ v_{k-l} & \Delta v_{k-l} & s_{k-l} \\ \vdots & \vdots & \vdots \\ v_{K-1-l} & \Delta v_{K-1-l} & s_{K-1-l} \end{bmatrix} \begin{bmatrix} \gamma_1 \\ \gamma_2 \\ \gamma_3 \end{bmatrix}. \quad (13)$$

Our previous work [21] has demonstrated that equilibrium driving condition is theoretically not possible to recover the model parameters due to the rank deficiency in X. In this article, we only consider non-equilibrium driving with time-varying inputs to avoid the potential parameter identifiability issue mentioned in [21]. Later in Section IV, we explore the parameter space to understand the sensitivity of model recovery with respect to the true parameter values.

IV. Parameter estimation on synthetic data

In this section we create synthetic ACC driving datasets and demonstrate that the RLS is able to accurately recover model parameters in the presence of delays and measurement noise. First we conduct a sensitivity analysis to explore the parameter space without measurement noise. This is done to understand experimentally the sensitivity of model calibration accuracy with respect to the true parameter values. Next, we investigate the effects of measurement noise on the performance of RLS. For both tests, we use the *mean absolute error* (MAE) between the simulated trajectories of the following vehicle using the estimated parameters and the true trajectories to measure the performance of RLS on model recovery.

A. Parameter identification using noise-free measurements

Considering *parameter identifiability* (whether the model parameters can be uniquely recovered from the input and output measurements) requires persistently exciting input signals [17], [18], we generate noise-free data based on a predefined, set of model parameters, i.e., the *true parameters*, and a known, non-equilibrium velocity profile of the lead vehicle. The states (velocity of the follower vehicle and the space gap in between) are updated through the discrete-time models of (2), (3) or (4). For each of the synthetically generated profile (*true trajectories*), a RLS is performed. This is done to understand at what regimes (parameter sets) is the accuracy of RLS estimator most sensitive to.

The data of ACC following trajectories are generated using a comprehensive set of model parameters. Specifically, we choose 8 equally spaced values of $k_1 \in [0.01, 0.1]$, $k_2 \in [0.08, 0.32]$ and $\tau_h \in [0.9, 2.3]$, and 5 equally spaced lag terms $\tau_a \in [0.1, 0.5]$ and delay terms $\tau \in [0.1, 0.5]$. The bounds of all the parameters are reasonable as found in [4].

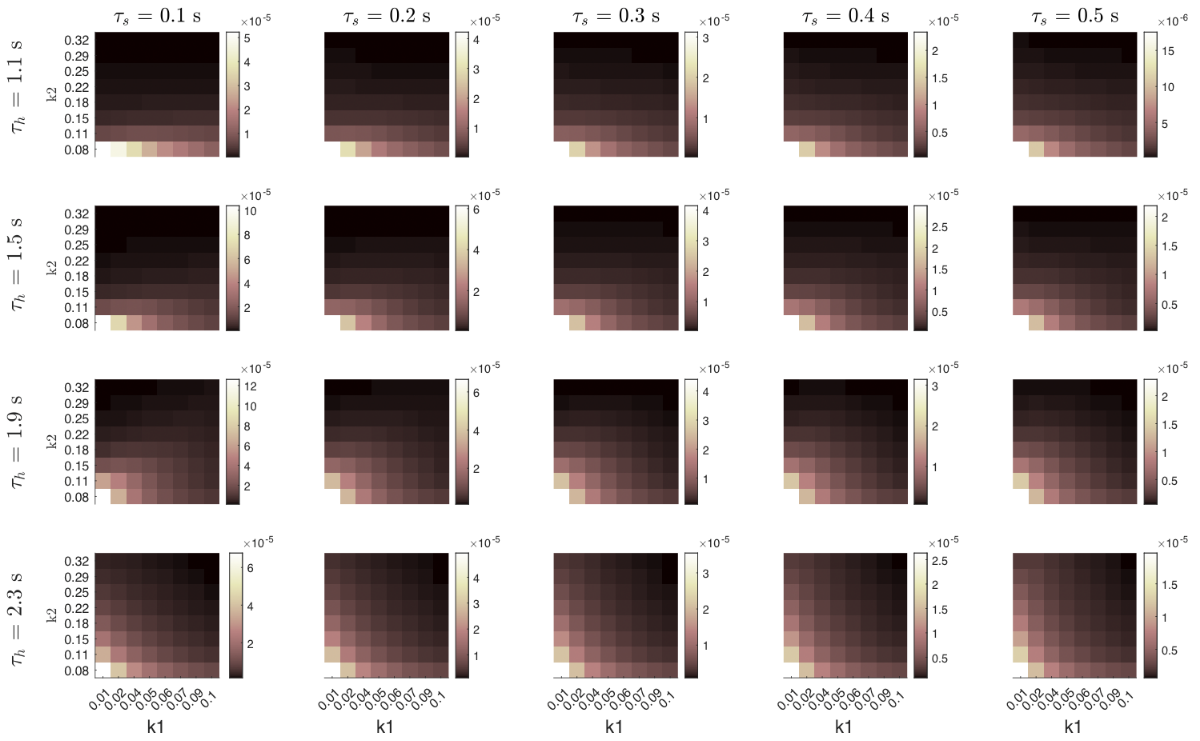

Fig. 1: RLS calibration accuracy under a comprehensive set of driving behaviors. The color scale shows the MAE of space gap between the simulated trajectory using the estimated parameters and the measured one. Each row corresponds to a fixed τ_h value and each column corresponds to a fixed delay value τ_s.

We consider all $8 \times 8 \times 8 \times 5$ possible combinations of parameter values, simulate the corresponding following vehicle trajectories, and use RLS to recover model parameters.

For all the considered regimes, RLS can exactly recover parameters of all three ACC models considered in this work. Figure 1 shows the sensor-delay model calibration performance (measured by MAE of the space gap) under the datasets parameterized by different values of k_1, k_2, τ_h and τ_s. For brevity, here we only show the estimation results of the DDE model (11), and note that the no-delay and actuator-lag model can also be exactly recovered with MAEs on the order of 10^{-5}. Note that, with fixed values of τ_h and τ_s, one can see that low gains k_1 and k_2 can result in a slightly larger estimation error; however, the effect on estimation accuracy is negligible since the errors are on the order of 10^{-5}m. This sensitivity analysis indicates that for all the regimes (parameter combinations) considered in this synthetic experiment, the parameters of all three models can be correctly identified with the RLS estimator.

B. Estimation under measurement noise

Next, we investigate the accuracy of model parameter estimation through RLS with the presence of measurement noise. Synthetic datasets are generated using fixed values of $k_1 = 0.08$, $k_2 = 0.12$, and $\tau_h = 1.5$, and the same velocity profile of the lead vehicle, but with various delays (8 equally spaced τ or $\tau_a \in [0.1, 0.8]$) and sensor noise levels. The sensor noise is assumed to be zero-mean Gaussian with various standard deviations on the position, velocity and the acceleration measurement of the tested vehicles. For convenience, we specify 1 unit of noise as equivalent to 0.1m, 0.05m/s and 0.05m/s^2 of standard deviation on the range (space gap or position), range rate (velocity) and acceleration measurements, respectively. These assumptions are based on the spec sheet of commonly used radar and GPS systems of ACC vehicles. As a reference, the GPS units used in [4], [7] are below 1 unit of sensor noise. In this study we consider the noise level varies between 1-8 units, which well covers the range and range rate accuracy of a significant number of on-board sensor devices.

Figure 2 shows the effect of sensor noise on the accuracy of calibrating a linear CTH-FL model without delays. It is observed that the MAE between the simulated trajectories using the RLS estimated parameters and the measurement data of both the space gap and the velocity monotonically increase with respect to the sensor noise level. The growth rate is slightly larger when the noise level is between 2-6 units.

Next we consider cases where both measurement noises

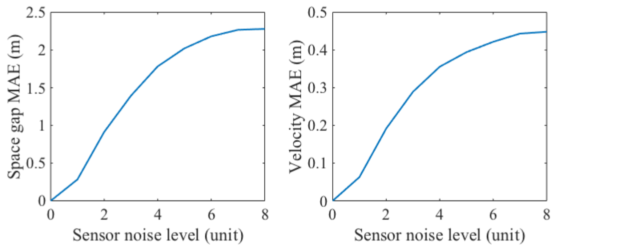

Fig. 2: Effect of sensor noise on model calibration error. 1 unit of noise corresponds to standard deviation of 0.1m and 0.05m/s of white noise on the position and velocity measurements, respectively.

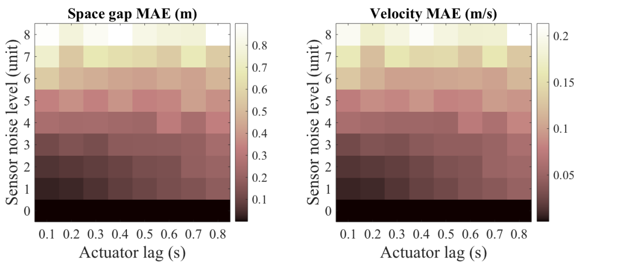

Fig. 3: Effect of sensor noise and actuator lag on model calibration error. 1 unit of noise corresponds to standard deviation of 0.1m, 0.05m/s and 0.05m/s^2 of white noise on the position, velocity and acceleration measurements, respectively.

and actuator lags exist. We synthetically generate ACC driving data given the same input signal (the velocity profile of the leading vehicle), but with various combinations of noises between 1-8 units and lags or delays between 0.1-0.8 second. The performance of RLS on recovering the actuator lag model is presented in Figure 3. One can see that the RLS estimator can exactly recover the model given noise-free measurement, which is consistent with the findings in Section IV-A. The estimator is robust when sensor noise is under 3 units and actuator lag under 0.3 second. When the noise is above 5 units, the accuracy is no longer sensitive to the lag contained in the dataset. Overall, the RLS estimator can well recover an actuator-lag model, with the fitting error significantly lower than the no-delay model does (70% reduced MAE in space gap and 60% in velocity).

Lastly, Figure 4 shows the effect of sensor noise and delays estimating the parameters of a sensor-delay model (11). The result shows that the RLS is robust under low sensor noise, but the performance is not consistent with high sensor noise. However overall, the MAE of space gap and velocity have approximately the same upper bound as fitting an actuator-lag model.

From a practical point of view, the presence of sensor noise can be a trivial concern because it has been shown that both

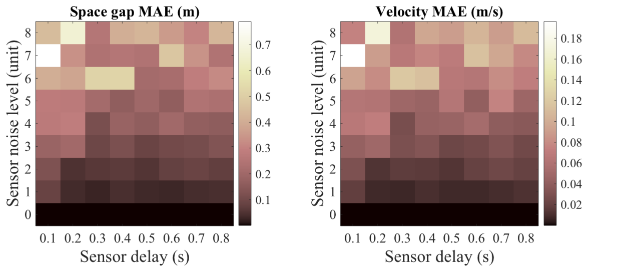

Fig. 4: Effect of sensor noise and delay on model calibration error. 1 unit of noise corresponds to standard deviation of 0.1m and 0.05m/s of white noise on the position and velocity measurements, respectively.

Criteria	No delay	Actuator lag	Sensor delay
Estimated parameter values	$k_1 = 0.0227$	$k_1 = 0.0227$	$k_1 = 0.0227$
	$k_2 = 0.1940$	$k_2 = 0.1940$	$k_2 = 0.1940$
	$\tau_h = 1.2272$	$\tau_h = 1.2272$	$\tau_h = 1.2272$
	-	$\tau_a = 0.4999$	-
	-	-	$\tau = 0.5000$
Running time (s)	11.477	17.314	15.209
MAE space gap (m)	2.0233	1.9960	2.0005
MAE velocity (m/s)	0.2384	0.2429	0.2410
MAE acceleration (m/s^2)	-	0.0784	-

TABLE I: Batch optimization on real ACC driving data.

the GPS and the ACC radar sensors contain measurement noise below 2 units. Under the RLS estimation routine, both the GPS and the radar units can provide data with high enough quality to ensure an accurate estimate of model parameters.

V. Case study on a 2019 ACC equipped vehicle

Now we proceed to test the RLS algorithm on the data collected from a 2019 ACC equipped vehicle. The experiment setup and data collection follow the same procedure described in our previous work [21]. Here, we additionally calibrate an actuator-lag model and a sensor-delay model in addition to the one without any delay terms using the RLS framework proposed in Section III.

Criteria	No delay	Actuator lag	Sensor delay
Estimated parameter values	$k_1 = 0.0174$	$k_1 = 0.0125$	$k_1 = 0.0171$
	$k_2 = 0.1641$	$k_2 = 0.2108$	$k_2 = 0.1665$
	$\tau_h = 1.2286$	$\tau_h = 1.2193$	$\tau_h = 1.2284$
	-	$\tau_a = 2.1362$	-
	-	-	$\tau = 0.1$
Running time (s)	0.0987	0.0917	0.8424
MAE space gap (m)	2.2451	2.3769	2.1937
MAE velocity (m/s)	0.2610	0.2818	0.2632
MAE acceleration (m/s^2)	-	0.0625	-

TABLE II: RLS estimation on real ACC driving data.

The estimation results are presented in Table II. Compared with a commonly used batch optimization shown in [7] (Table I), RLS improves the running time more than 100x with a slight compromise in accuracy (less than 0.3m in spacing and 0.03m/s in velocity MAE). The sensor-delay model produces trajectories with the highest fitting accuracy (2.19 m in space gap and 0.26 m/s in velocity, respectively). Overall in this case study, all three models are approximately equivalent from a pure data-fitting perspective.

From this particular case study, we find that RLS can successfully recover the linear ACC models assuming the presence of actuator lags or sensor-delays. The captured behavior of the tested ACC vehicle is shown to provide enough information to identify the model parameters. Model calibration using RLS on different datasets can also be verified. We provide supplementary ACC driving data collected in previous studies [3], [4], [7], [21], including 10 different commercially available ACC-equipped vehicles and various input profiles. Readers are encouraged to download from `https://acc-dataset.github.io/datasets/`, and try the proposed RLS framework for model calibration.

VI. Conclusion

In this work we extend a recursive least squares algorithm to calibrate ACC models with systematic delays including sensor delays and actuator lags. We show that RLS is able to exactly recover model parameters under noise-free setting. In addition, the RLS estimator is robust to sensor noise of the level of on-board GPS and radar devices. Finally, the method is tested on real ACC driving data collected from on-board radar devices, and the results show that the performance of an RLS estimator is well-matched to that of a commonly used batch optimization technique (about 2m MAE in space gap and 0.24m/s MAE in velocity), with a runtime that is of approximately two orders of magnitude faster.

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