Data-driven methods for dockless bike infrastructure planning

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***Abstract*—In this work we address the problem of data-driven placement of critical bike infrastructure to address user route demand. The proposed framework uses trip origin-destination data that is commonly produced by bike share operators and included in standard data feeds. It does not require intermediate trip GPS data, which is not yet widely available. To compensate for lack of full path information in the data, the proposed method estimates the path of each trip using a shortest path routing algorithm, which runs on a bike-accessible network graph with user preferences. We apply this path estimation to each trip, which results in a volume estimate on each edge in the network graph. We can then assess the effectiveness of infrastructure upgrades from trip distance coverage and user impact perspectives. The framework is applied on a data set of tens of thousands of dockless bike share trips that occurred during the first month of a pilot program with ofo bikes at Vanderbilt University. Case study findings demonstrate that approximately 40% of observed trip distance can be covered by improving only 5% of the infrastructure and 75% of trips will travel some portion of their path on the upgraded infrastructure. This highlights the significant potential benefits for a modest infrastructure investment. While the capture rates may be different on each network, the methodological tools are applicable to data from any docked or dockless bike share system and infrastructure network. The tools will serve as the basis for a software platform that will help any city analyze data collected from shared use mobility devices including bikes and electric scooters and assess infrastructure investment.**

***Index Terms*—data, transportation, bicycle, infrastructure, planning**

I. Introduction

Due to the health benefits for riders, the benefits for communities, and the reduction of harmful impacts of car use, increasing the rate of bicycling is achieving significant investment [1]. The need for dedicated biking infrastructure (e.g., bike lanes, off-street trails, turn boxes, and signals) is motivated by the increased safety and comfort for cyclists [2]. Infrastructure investments are important both for existing users and increasing rates of cycling by inducing demand from reluctant cyclists [1], [3], [4].

The planning problem of identifying the need and precise location for dedicated bike infrastructure is multidimensional and ultimately depends on many factors external to cyclists themselves that include existing road and sidewalk infrastructure, cost, safety, and politics [5], [6]. These considerations can be predicated upon the estimation or measurement of the number of bicycle trips that will be impacted by the infrastructure. This quantity is often difficult to measure and traditional techniques involve user surveys [1], [7], [8] and data collection from GPS [3], [4], [6].

Compared to cyclists who own their bikes for commuting or leisure, the prevalence of docked and new dockless bike share systems create opportunities to collect and analyze data on a very large scale [9]. Docked bike share systems are comprised of fixed dock locations where users must pick up and drop off bikes, and operate in many major cities throughout the world. Most docked bike share systems only record data on the origin and destination station of each trip [10]. Dockless bike share systems are a more recent operational model where users find and unlock an available bike and may drop the bike off anywhere at their destination. These systems record GPS data of origin and destination locations and some may record GPS data at regular intervals from the bike or user's smartphone [6]. However, the GPS data can have very low temporal resolution, if it is reported at all.

Surveys of commuter and leisure cyclist behavior have shown that cyclists are willing to travel a longer route utilizing dedicated bike infrastructure, compared to a shorter route without dedicated infrastructure. This increased route length has been estimated up to 20% for off-street bike trails [8] and 10% for on-road bike lanes [11], [12]. Due to the nonzero cost of installing bike infrastructure, it is desirable to efficiently install new bike infrastructure to serve as many cyclists as possible. The effectiveness of a bike network is thereby a function of the network topology and the distribution of trips and volume around and through the network.

This work addresses the problem of data-driven placement of bike infrastructure based on network topology and distribution of observed trips. The objective is to find the volume of trips on a network segment, which has the notable difficulty of data sparsity with regard to bike trip data. The framework we develop requires only origin-destination data gathered from shared bike systems. It estimates trip paths by shortest accessible path routing between trip origin and destination on the exact infrastructure network. Trip paths are aggregated to estimate volume at all points on the network and assess the impact of infrastructure upgrades. The framework can also take into account induced path changes as a result of new infrastructure and user preferences. This framework is straightforward to implement and useful for cities with a large volume of bike share data to evaluate existing and potential bike infrastructure upgrades. We illustrate the effectiveness through a case study on bike share data at Vanderbilt University, which is an example of the canonical last-mile setting (i.e., on a college campus).

To our knowledge, the only other work on the problem of data-driven bike infrastructure assessment and planning is that of Bao et al., which used GPS trajectory data collected from over 230,000 shared dockless bikes in Shanghai, China [6]. The placement of bike infrastructure is posed as an optimization problem, where constraints include budget, construction, and utilization.

The problem of identifying links in a network to upgrade is a common issue facing planners and governments [5], [6]. It is closely related to work in network resilience where links are similarly identified for investments to reduce risk and vulnerability of the network. The study of transportation resiliency can be viewed from graph theoretic perspective based on network topology, supply, and demand [13], [14].

The remainder of this paper is organized as follows. First, we describe the methodology used in the proposed framework. Second, we present the details of the dockless bike share pilot program and case study at Vanderbilt University. Third, we present the results of applying the framework to the case study, including the volume estimation and assessment of potential infrastructure upgrades. Finally, we discuss the results in the context of both the case study and general bike planning, including limitations and practical implications; we also outline future work on this framework and platform.

II. Methodology

In this section, we detail the methodology in the proposed framework, which is designed to use bike share trip origin-destination data to estimate the volume on each segment of the network. Precisely, this includes processing of bike share data, construction of a bike-accessible network graph, routing of trips across the graph, and assessment of potential infrastructure upgrades.

A. Bike share data

Each record in the raw bike share data is a single trip that includes a unique trip ID, ID of bike used, origin timestamp (local time zone), origin location (latitude, longitude), destination timestamp, and destination location. From this, we can place the trip spatially in the network and/or match the origin and destination points to specific predefined locations (e.g., bike docks, designated parking). Origin-destination data is used to filter out trips that contain obvious data errors. For example, timestamps at each origin-destination point are useful for identifying and removing outliers that report unreasonable average trip speeds.

B. Network graph, routing, and cost function

We construct a directed, weighted graph for a bike accessible network, $G = (V, E)$, made up of vertices, V, and edges, E. Edges represent existing infrastructure, such as roads, sidewalks, or crosswalks, across which bike trips can be routed depending on local regulations. Vertices are simply the intersections between edges at logical points of delineation, such as a road intersection or the termination point of a sidewalk.

The process of associating trip points to a network graph is referred to as *map matching*. Origin-destination coordinates are associated with the closest vertex in the graph computed by Euclidean distance. The closest vertex must be, at maximum, 150 feet from the origin or destination point, which accounts for parking away from paths or high GPS error caused by cover or buildings. In the case that one or both of the points does not match a graph vertex, the pair is disregarded.

A shortest path routing algorithm finds a shortest path between two graph vertices such that the sum of edge costs forming a path between vertices is minimal. In the present work, shortest path routing for a bike trip between the corresponding vertices on the infrastructure graph is done using Dijkstra's algorithm [15]. Dijkstra is the seminal algorithm used for shortest path routing for a graph with positive edge costs, and is typically sufficient for small graphs. There have been many enhancements to Dijkstra's algorithm (e.g., A*, bidirectional search, and other modern routing algorithms) [16] that are applicable for large graphs or large amounts of trips.

We construct a generalized cost function for graph edges that quantifies in real units the time required to traverse the edge and takes into account factors for bicycle accessibility and user routing preferences. Edges in the graph are labeled as accessible by bike based on whether or not they spatially intersect with any obstructions (e.g., stairs). User preferences are accounted for by penalizing edges that intersect roads, where waiting time and discomfort may be associated with crossing. This function, for edge i, takes the form

$$c_i = \frac{l_i}{v_{\text{bike}}} + r_i t_{\text{road}} + s_i t_{\text{stair}}, \quad (1)$$

where c_i is the cost assigned to edge i in terms of time cost to traverse, l_i is the length in feet of edge i, v_{bike} is the bike velocity in feet per second, r_i is an indicator variable that is equal to 1 if edge i crosses a road and 0 otherwise, t_{road} is the additional time cost in seconds for edges crossing a road, s_i is an indicator variable that is equal to 1 if edge i crosses a road and 0 otherwise, and t_{stair} is a large time penalty in seconds to heavily penalize edges intersecting stairs. This work assumes reasonable values for t_{road} and t_{stair} but these values could be chosen based on a particular locale or scenario. This cost function makes the notable assumptions that 1) all user operate at the same speed, v_{bike}, which we later assign to be the mean speed of trips in the dataset, and 2) speed over each trip path is uniform; these assumptions are addressed further in Section VI.

C. Induced path changes

The consideration also exists that volume is elastic to the built environment and that cyclists will take a longer path with respect to length in order to use upgraded bicycle infrastructure (e.g., bike lanes, off-street trails, turn boxes, and signals). As discussed in literature in the Introduction section, cyclists will ride 10% and 20% longer routes to use upgraded infrastructure [8], [11], [12]. We therefore take $\delta > 0$ to be the relative increase in cost that cyclists will ride, and assign values $\delta = 10\%$ and $\delta = 20\%$. We can equivalently reduce the cost of upgraded infrastructure by the factor $\frac{1}{1+\delta}$ because under this assumption an upgraded link of length $l_i(1+\delta)$ is equivalent to a non-upgraded link of length l_i. The resulting cost (which now represents a real time cost with a user experience factor), c_i^{improved}, of an upgraded edge is

$$c_i^{\text{improved}} = \frac{\frac{l_i}{v_{\text{bike}}} + r_i t_{\text{road}} + s_i t_{\text{stair}}}{1+\delta}. \quad (2)$$

D. Infrastructure upgrades

The given problem of upgrading infrastructure such that user impact is maximized can be posed as an optimization problem, such as in [6]. Here, we perform *greedy assignment* of infrastructure upgrades on network graph edges according to trip volume, where the edge with the highest estimated trip volume is the first edge upgraded. Likewise, the second edge to receive an upgrade is the edge with the second highest estimated trip volume. We would expect that beginning with the highest volume edges, the addition of upgraded infrastructure to each would impact a relatively large share of trips. As we upgrade infrastructure on edges with lower volume, the trips impacted would gradually diminish until the least used edge gains little or no additional benefit.

In the case of induced path changes, the routing of all trips must be reassessed after each infrastructure upgrade. This is due to the fact that an edge weight is reduced each time an edge is upgraded and this may change some trip paths.

III. Dockless bike share case study

In this section, we introduce the dockless bike share pilot program at Vanderbilt University, present some descriptive statistics of the data gathered, and discuss the network graph constructed for campus.

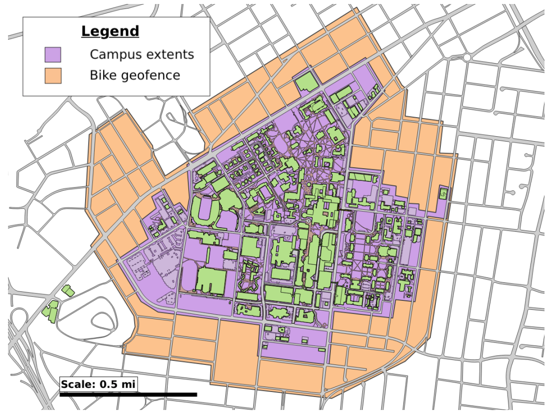

Fig. 1: Geofence set up for shared bikes (orange outer boundary) that encourages rides to stay within Vanderbilt jurisdiction. The approximate campus extents are shown in purple with campus facilities in green.

A. Pilot program

In March 2018, Vanderbilt University began a six-month pilot program with ofo, an international dockless bike share operator. The pilot program complements the goals of the University's land use strategy and transportation strategy, both of which endeavor to increase the share of commute alternatives to single occupancy vehicles. The timing of the pilot program coincides with the evaluation and regulation of similar dockless shared use mobility systems by cities across the country. In the first month of operation, the pilot program was free for Vanderbilt users and generated tens of thousands of rides. The exact number of rides is confidential ofo information and for this reason, descriptive statistics and results are presented normalized by this total number of trips within the first month of the pilot.

Included in the pilot program agreement was the establishment of a boundary referred to as a *geofence*. Users were encouraged to keep their rides within the geofence. An overlay of the ofo geofence (orange) on the campus boundary (purple) is shown in Figure 1. The geofence proved effective in containing trips, with 95% beginning and ending within the boundary.

B. Descriptive trip statistics

The primary data product from the pilot program is the origin-destination location coordinates and start and end time of each individual trip. Through this data, we are able to see that the vast majority of trips were quite short. They had a mean distance of 0.41 miles and mean duration of 6 minutes. For reference, a trip from some primary on-campus housing locations to the core of the academic campus is about 0.6 miles and the full width and length of campus are both approximately 0.9 miles. Trips with duration of less than two minutes and greater than 15 minutes were almost exclusively anomalous and regarded as outliers. Mean speed of all trips was 4.3 mph and, likewise, trips with average speed less than 1 mph or greater than 20 mph were removed as outliers. The

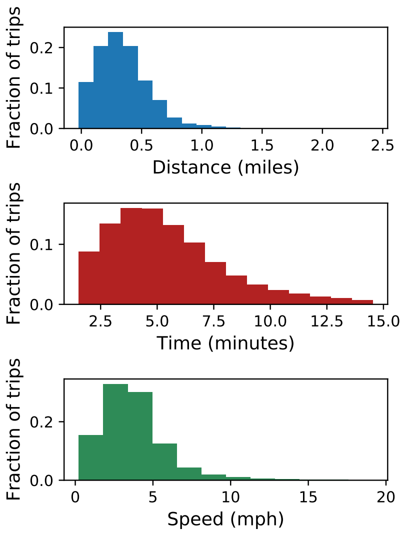

Fig. 2: Distributions of trip distance, duration, and speed (top to bottom) for the first month, with outliers removed.

distribution of trip distances, times, and speeds with outliers removed are shown in Figure 2.

C. Campus network graph

The routing graph for the campus infrastructure is constructed by processing up-to-date GIS data layers for paths and sidewalks into the graph edges. We assume that cyclist routing on Vanderbilt's campus for this case study will involve a negligible amount of road use and routing along roads can be handled by adjacent sidewalks, so roads with parallel sidewalks are processed into one edge in each direction.

The edge cost c_i has units of seconds. We set $v_{\text{bike}} = 6.3$ to be the mean trip speed in feet per second, $t_{\text{road}} = 120$ to be a reasonable time cost (in seconds) of waiting at an intersection, and $t_{\text{stair}} = 900$ such that routes cross stairs only with no other reasonable option. Alternatively, edges intersecting stairs (i.e., $s_i = 1$) could be removed from the graph, entirely.

User preferential routing that accounts for the effects of crossing roads manifests itself commonly in the data on two pedestrian bridges that cross a major road through part of campus. One of these bridges is highlighted in yellow in Figure 3 with the vertices (light blue points) and edges (blue lines connecting points) of the overlaid network graph. The edge crossing the road at street level (grey) has the time penalty applied, while the route crossing the bridge (red line) is not penalized. It is observed that the overwhelming majority of cyclists or pedestrians cross this location on the bridge if not required to go to street level based on their destination.

IV. Results

In this section we explore results of upgrading infrastructure related to three cases: a *base case* where all trips are routed along their shortest path using edge weights defined by equation (1) and routes do not change; and two *induced path change cases* where the cyclists will change their routes

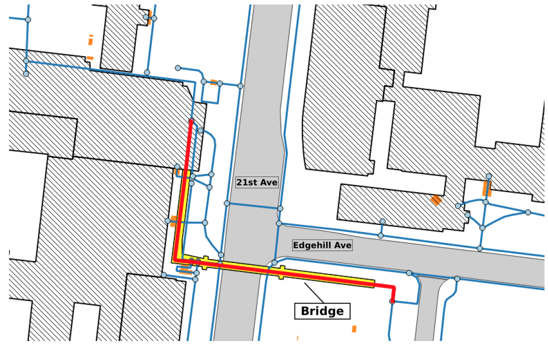

Fig. 3: The routing graph is comprised of vertices (light blue points) and edges (blue lines). Edges cross both a pedestrian bridge (yellow) and the road beneath the bridge (grey). The edge crossing only the road will incur a penalty to encourage routing across the bridge (red line). Stairs are shown as small orange lines.

and travel 10% or 20% further to use upgraded infrastructure, according to edge weights defined by equation (2). We first present the base case volume results. We then look at the two cases that analyze the impact of trip path changes induced by upgraded infrastructure. The computational requirements of the framework are also addressed.

Using the estimated volume, n_i, across each edge, we define the *bike distance traveled* across edge i as d_i. Bike distance traveled is expressed as a fraction of total bike distance observed such that $d_i = \frac{n_i l_i}{\sum_1^k m_k}$, where n_i is the number of trips crossing edge i, l_i is the length of edge i, and m_k is the length of trip k. We also define two metrics to quantify the effectiveness of upgrading bike infrastructure to the network:

- *trip distance covered* - fraction of total bike distance traveled that occurs on upgraded infrastructure
- *trips impacted* - fraction of trips that travel any amount of their estimated route on upgraded infrastructure.

The *capture rate of trip distance* and *capture rate of trips impacted* are defined as the rates at which trip distance covered and trips impacted increase with the incremental addition of upgraded infrastructure.

A. Base case

Following the calculation of the shortest path for each trip according to the cost function, equation (1), we tabulate the number of trip paths crossing each edge, i, in the network to be n_i. The base case edge-level volume is shown by map in Figure 4. Visual inspection of Figure 4 reveals two separate corridors of concentrated volume, colored yellow on the map.

We analyze the effect that increasing the amount of upgraded infrastructure has on trip distance covered and trips impacted by greedy assignment of upgraded infrastructure to graph edges in the order of maximum estimated volume, as described in the Methodology section. It follows that trip distance covered and trips impacted will both increase continually to capture all observed trip mileage and all trip occurrences until all edges are upgraded. The blue line in Figure 5 shows the

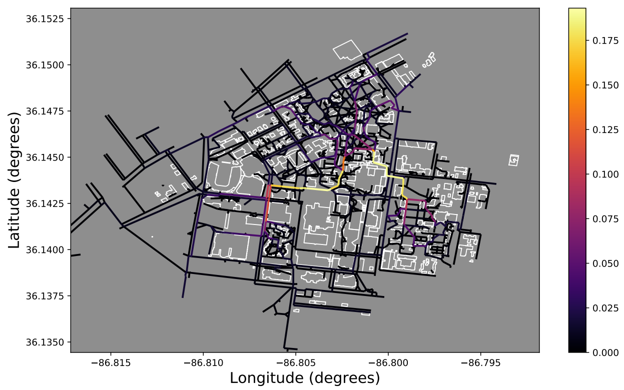

Fig. 4: Total estimated volume on each graph edge as a proportion of total trips observed.

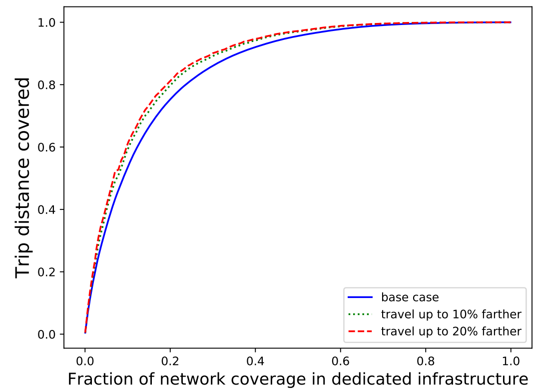

Fig. 5: Fraction of trip distance covered under the base case (blue line) compared to cases where cyclists are willing to travel 10% or 20% further to use upgraded infrastructure (green dotted and red dashed lines, respectively).

TABLE I: Trip distance covered by fraction of infrastructure upgraded (in terms of length) on the network, in the base case and two cases of induced trip path changes.

	2% network upgraded	5% upgraded	10% upgraded	25% upgraded
$\delta = 0\%$ (base case)	21%	37%	55%	81%
$\delta = 10\%$	23%	40%	60%	85%
$\delta = 20\%$	24%	42%	62%	87%

effect on trip distance capture by upgrading infrastructure in the base case. The first infrastructure upgrades result in large gains in trip distance covered, but there exists a clear point of diminishing returns. We see that upgrading just 5% of the network covers 37% of the total trip distance and upgrading 10% of the network covers 55% of total trip distance. This is notable because even with no induced routing change (i.e., preference for routes with infrastructure upgrades), a very large amount of trip distance can be captured on upgraded infrastructure with extremely low coverage of the network.

B. *Induced path change cases*

Next we consider the cases where upgraded infrastructure is likely to induce some marginally longer trips to change paths to use upgraded bike infrastructure. We reduce edge cost for upgraded infrastructure according to equation (2) using values $\delta = 10\%$ and $\delta = 20\%$, as explained in Section II.

Unsurprisingly, inducing trips to use upgraded infrastructure increases the trip distance covered at every amount of infrastructure improved, as we can see in Figure 5. The redistribution of trip paths with induced use of upgraded infrastructure increases the trip distance covered to 42% with 5% of network upgraded and 62% of trip distance covered by 10% of upgraded infrastructure. Looking at the number of trips impacted, the trend is even more dramatic: about 75% of trips travel some portion of their route on upgraded infrastructure after it covers only 5% of the network. A more thorough tabulation of trip distance covered and trips impacted for various levels of network infrastructure improvement are included in Tables I and II, respectively.

TABLE II: Trips impacted by fraction of infrastructure upgraded (in terms of length) on the network, in the base case and two cases of induced trip path changes.

	2% network upgraded	5% upgraded	10% upgraded	25% upgraded
$\delta = 0\%$ (base case)	57%	74%	85%	90%
$\delta = 10\%$	58%	75%	86%	91%
$\delta = 20\%$	60%	76%	86%	91%

C. *Computational requirements*

Computation of shortest path routes for the network in the Vanderbilt case study required approximately one minute per 10,000 trips on a single threaded implementation of the routing on a 3.6 GHz 16-core CPU. For the pure shortest path routing and assessment of upgraded infrastructure in the base case with no behavior change, shortest path computations need only performed once. However, when routing behavior changed because of reduced costs on edges with upgraded infrastructure, shortest paths must be computed at each installation level. Approximately 200 installation levels were assessed for each value of δ. This resulted in a runtime of approximately 15 minutes per 10,000 trips per value of δ. Computation time could be sped up by a more efficient routing algorithm, but in the case study gains would be minimal due to the graph size.

V. Discussion

The topology of the Vanderbilt University campus and the distribution of trip origins and destinations creates definitive bottlenecks that are easy to identify from the estimated paths. Even in the base case, the magnitude of this coverage is dramatic and shows just how little bike infrastructure may be sufficient to cover a majority of trips. In other areas of campus, volume is more sparsely distributed. Capture rate of trips and trip distance is much slower for these areas and would require significantly more infrastructure to be upgraded.

Under the assumption of induced trip path changes, the capture rate for the case study is higher and leads to additional coverage of 5% of trip distance at the 5% level of network improvement. We believe this increased capture rate is realistic. Given the absolute size of Vanderbilt's campus and its heavily pedestrian nature, it is reasonable for cyclists to travel slightly longer routes to achieve a clear path through congested areas.

The trips impacted by the infrastructure is only slightly higher under induced path routing, but the rate of trips impacted is already very high. This rate is significant because it promotes the positive culture change associated with alternative mobility [1].

There exist a few notable limitations of this work. We rely heavily on 1) the assumption that origin-destination volume is indicative of path volume between points; that is, cyclists are riding from their origin to destination without intermediate stops or sightseeing; and 2) that user preferences for routing and riding speed are uniform across trips and across the path (i.e., they are not influenced by exogenous factors). Additionally, the Vanderbilt network topology is unique and the distribution of trips creates notable bottlenecks. This concentration of volume may not extend to other networks. Lastly, but importantly, the issues of system equity are not considered. We assume that rebalancing of bikes occurs to address equity considerations and geographical bias.

VI. Conclusion

We present in this work a framework by which origin-destination data from bike share systems, or otherwise, may be evaluated in the context of existing and potential upgraded bike infrastructure. In this manner, we can identify critical infrastructure segments for improvement and the ridership coverage that will be associated with them. The framework accommodates an induced volume factor where cyclists will travel further to use upgraded infrastructure. The framework consists of ingestion and exploration of origin-destination data, routing of individual trips across a network graph of existing infrastructure, and the evaluation of infrastructure improvements on estimated paths taken by cyclists.

Using the case study from Vanderbilt University, we test the framework and find dramatic distributions of trip volume. Investment in only 5% of the campus infrastructure can likely cover at least 37% of total trip distance. This is without the effect of induced infrastructure use, which could result in the same investment in 5% of the infrastructure yielding 42% of trip distance covered. At this level of infrastructure improvement, up to 76% of trips could use the upgraded infrastructure to some extent during trips.

Future work includes extending to personalized routing with different user speeds and non-uniform speeds across a trip path. For example, this could take into account topography of the route, real-time traffic congestion of the area, and condition of existing infrastructure. Larger networks are also of interest, which could include docked bike share systems or other dockless devices such as scooters. We also plan to incorporate OpenStreetMap (OSM) into the framework so that it may be trivially extended to any city.

The framework is entirely generalizable and potentially useful for cities and other entities that are interested in using data streams from shared mobility devices that are established and/or gaining popularity. The evaluation of infrastructure in this manner helps users of these mobility platforms and other users in the system who walk or own their own bike.

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